

# Heuristic Approach to Problem-solving Skills and Mathematics Performance of Humanities and Social Science Learners

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**Abstract**— *This quasi-experimental study aimed to determine the effectiveness of the heuristic approach on problem-solving skills and mathematics performance of Grade 11 Learners. Eighty (80) Grade 11 learners were the participants of the study. The experimental group of 40 learners were exposed to the heuristic approach as an intervention, while the control group of 40 were exposed to the traditional method. Data were gathered using a validated researcher-made the 10-item problem-solving test and a 50-item multiple-choice test. The data gathered were analysed and interpreted using appropriate statistical tools. Based on the findings, the pretest problem-solving skills of the learners with or without the heuristic approach were “developing” while the mathematics performance was “approaching proficiency”. The posttest problem-solving skills of the learners with or without the heuristic approach were “approaching proficiency”, while the mathematics performance was “proficient”. No significant difference was found between the pretests. Still, there was a significant difference in the posttest problem-solving skills and performance in mathematics of the learners with and without the heuristic approach. There was a significant difference between the pretest and the posttest problem-solving skills and performance in mathematics of the learners with and without the heuristic approach. The findings significantly affected the learners’ problem-solving skills and mathematics performance using a heuristic approach.*

**Index Terms:** *Effectiveness of Heuristic approach, Problem-solving, Mathematics performance, Humanities and Social Science learners*

## I. INTRODUCTION

Problem-solving has long been recognised as one of the hallmarks of mathematics. One of the most significant goals of mathematics education is to have students become good problem solvers (Billstein, Libeskind & Lott, 2012). According to Krulik and Rudnick (2016), problem-solving is how an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. The process has been analysed and can be represented as a series of steps referred to as a heuristic plan or heuristics. Heuristics provide a "road map"; a blueprint that directs one's path toward a solution and resolution of a problem.

Krulik and Rudnick identified eight strategies applicable to mathematical problem solving: 1. Computing or Simplifying; 2. Using a Formula; 3. Making a Model or Diagram; 4. Making a Table, Chart, or List; 5. Guessing, Checking, and Revising; 6. Considering a Simpler Case; 7. Eliminating; and 8. Looking for Patterns.

George Pólya's problem-solving methodology has been widely applied across disciplines in real-world problem-solving scenarios. Here are a few examples:

**1. Engineering:** Engineers often encounter complex problems that require systematic problem-solving. Pólya's approach has been used to devise strategies for tackling engineering challenges.

**2. Medicine:** Medical professionals face complex

diagnostic and treatment challenges. Pólya's problem-solving approach can be applied in medical decision-making processes.

**3. Business and Management:** Pólya's problem-solving approach can be utilised in various business and management scenarios.

**4. Education:** Pólya's problem-solving methodology has been widely used in mathematics education. Teachers often employ this approach to guide students in solving mathematical problems. Students are encouraged to understand the problem, devise a plan, execute it using relevant mathematical concepts, and reflect on their solution. This approach helps develop students' critical thinking, problem-solving, and mathematical reasoning abilities.

**5. Research and Scientific Inquiry:** Pólya's problem-solving approach is also relevant in scientific research.

This paper seeks to determine how a heuristic approach can improve the problem-solving skills and mathematics performance of Grade 11 learners in Pototan National Comprehensive High School.

## II. LITERATURE REVIEW

### A. Heuristic Approach

A heuristic strategy ignores part of the information, intending to make decisions more quickly, frugally, and/or accurately than more complex methods (Gigerenzer & Gaissmaier, 2011).

The most fundamental heuristic is trial and error, which can be used in everything from matching nuts and bolts to finding the values of variables in algebra problems.

The term heuristic, from the Greek, means "serving to find out or discover" (Todd & Gigerenzer, 2012). In the context of problem-solving, heuristics are experientially derived cognitive "rules of thumb" that serve as guides in problem-solving processes. Heuristics guide problem solvers by helping them simplify choices regarding the numerous immensely complex and imperfectly understood factors that act simultaneously to shape problems.

Here are eight different problem-solving approaches or heuristics that individuals can employ:

**1. Trial and error:** This approach involves attempting different solutions or strategies through trial and error until a successful outcome is achieved.

**2. Working backwards:** This heuristic involves starting from the desired goal or solution and working backwards to determine the steps needed to reach that goal.

**3. Breaking the problem:** This approach involves breaking a complex problem into smaller, more manageable sub-problems.

**4. Using analogies:** This heuristic involves drawing parallels or connections between the current problem and similar problems that have been solved before.

**5. Making a guess and refining it:** This approach involves making an initial guess or hypothesis and then refining it through iterations or adjustments based on feedback or additional information.

**6. Using a systematic algorithm:** This heuristic involves following a step-by-step procedure or algorithm to solve a problem.

**7. Drawing diagrams or visual representations:** This approach involves creating visual representations, such as diagrams, charts, or graphs, to understand the problem better and visualise potential solutions.

**8. Working with a partner or in a group:** Collaborative problem-solving can involve brainstorming ideas, discussing different approaches, and collectively working towards a solution.

### **B. Benefits of Heuristic Approach**

Heuristic methods are rules of thumb for making progress on complex problems (Polya, 1973). They are general suggestions on strategies to help solve problems (Schoenfeld, 1985). According to Bruner (1960), they are methods and techniques that can be helpful in problem-solving. In sum, heuristic methods can be explained as non-rigorous methods of achieving solutions to problems, ideas that have been useful in previous problem-solving and can be applied to solve our current problems.

A heuristic is a mathematical problem-solving strategy formulated in a free-of-context manner and done systematically. Moreover, a heuristic approach can

encourage connecting mathematical thoughts by examining exceptional cases, drawing a diagram, specialising the solution, and generalising the solution (Hoon, Kee, and Singh, 2013). It is associated with non-routine mathematical problems such as looking backwards or thinking forward. Several studies were conducted to improve students' skills in solving mathematics problems. Hoon, Kee, and Singh (2013) investigated students' responses to applying the heuristics approach in solving mathematical tasks and their abilities in applying the heuristics approach. Reiss and Renkl (2012) proposed using heuristic worked-out examples in proving. They suggested that this should be integrated into mathematics classrooms frequently so that students will learn to extract needed information from the problems. Novotná (2014) aimed to improve the pupils' problem-solving culture by dealing with strategies such as analogy, guess-check-revise, problem reformulation, solution drawing, systematic experimentation, way back and use of graphs of functions. The studies show how strategies can improve mathematics problem-solving; Koichu, Berman, and Moore (2014) aimed to promote heuristic literacy in a regular mathematics classroom.

Here are some of the benefits of using the heuristic approach: (a) Speed: Heuristics allow us to make decisions quickly, which is beneficial in time-sensitive situations (Gigerenzer & Gaissmaier, 2011), and (b) Efficiency: They reduce the cognitive load and simplify decision-making processes, making them more efficient (Shah, & Oppenheimer, 2008). (c). Practicality: Heuristics are practical tools that can be used in everyday life, not just in academic or professional settings (Todd & Gigerenzer, 2001).

### **C. Problem-solving Skills**

Problem-solving skills are essential in mathematics. Here are some key problem-solving skills that are particularly relevant to mathematics:

- a) Analytical thinking: Mathematics requires analysing problems and breaking them down into smaller, more manageable parts.
- b) Creative and critical thinking: Mathematics often involves finding innovative and alternative approaches to problem-solving. Creative thinking allows for the exploration of different strategies and perspectives. In contrast, critical thinking enables evaluating solutions for accuracy and logical reasoning.
- c) Logical reasoning: Mathematics is based on logical principles and deductions. Developing strong logical reasoning skills helps construct proofs, identify valid arguments, and make logical connections between concepts and ideas.
- d) Persistence and patience: Mathematics problems can be complex and require perseverance.
- e) Problem decomposition: Breaking down complex

problems into simpler, more manageable components is a valuable skill in mathematics.

- f) Visualisation: Visualising mathematical concepts, relationships, and solutions can aid problem-solving.
- g) Pattern recognition: Recognising patterns and identifying regularities in mathematical problems is valuable.
- h) Flexibility: Mathematics problems often have multiple approaches and solutions. Being flexible in problem-solving allows for the exploration of different methods and the ability to adapt strategies as needed. Developing and honing these problem-solving skills can significantly enhance one's ability to tackle mathematical challenges effectively. Regular practice, exposure to a variety of problem types, and seeking guidance from teachers or mentors can further strengthen these skills

#### **D. Mathematics Performance**

Quality mathematics education should enable learners to form a positive and appropriate image of mathematics. It must be faithful to mathematics in its content and practices to be possible. It should allow learners to understand which needs are met by the mathematics they are taught and that mathematics forms part of a long history linked to humanity.

Many learners find mathematics difficult. Learners' performance in this subject has been poor at all levels, as indicated by the results of the National Achievement Tests for both elementary and secondary levels. According to the latest Gallup youth survey conducted in 2004 (Saad, 2005), mathematics is the subject that teenagers find most challenging in school. Thus, Saad (2005) said that it is not surprising that the subject has the lowest performance rate.

In the Philippine context, as cited in research conducted by Cabahug and Ladot (2005), the University of the Philippines' greatest failure is in mathematics. Furthermore, Cabahug and Ladot (2005) said that the only significant factor for students required to take Mathematics 11 was their attitude towards the subject.

The National Assessment of Educational Progress (NAEP) assesses student performance in mathematics in grades 4, 8, and 12 in public and private schools nationwide. NAEP mathematics scale scores range from 0 to 500 for grades 4 and 8 and 0 to 300 for grades 12. NAEP achievement levels define what students should know and be able to do: Basic indicates partial mastery of fundamental skills, Proficient indicates demonstrated competency over challenging subject matter, and Advanced indicates superior performance beyond proficient.

According to Maliki, Ngban, and Ibu (2009), mathematics affects all aspects of human life to different degrees. According to the National Mathematics Advisory Panel (2008), mathematics is used daily.

On several accounts, mathematics teachers were judged as

the primary determinant in the success or failure of students in the subject. This scenario is actual for many reasons; according to Idowu (2015), they are the primary custodians of students, and how and how they perform this role is essential. Because of this, mathematics teachers are expected to know what is necessary for teaching and the ability to disseminate such knowledge appropriately, resulting in learning. According to Soer (2009), mathematics teachers should be able to communicate the required knowledge in a clear, informative and precise manner to their students. Unfortunately, according to Okafor and Anaduaka (2013), mathematics teachers do not do this. According to both researchers, most teachers are not ready to go the extra length in their teaching.

In a study by Avong (2013), a shortage of qualified mathematics was judged to be the most contributing factor to participants' poor performance in a remote area of Kaduna state. Teachers' attitudes were also linked to students' poor performance in a study conducted by Osunde and Izevbogie (2006).

### **III. PURPOSE OF THE STUDY**

This study aimed to determine the effect of a heuristic approach on the problem-solving skills and mathematics performance of Grade 11 learners in Pototan National Comprehensive High School, Pototan, Iloilo, during the school year 2024-2025.

Specifically, this study sought answers to the following questions:

1. What is the level of problem-solving skills of the learners before and after their exposure to the heuristic method approach in teaching and lecture method in teaching?
2. What is the mathematics performance of the learners before and after their exposure to the heuristic approach in teaching and lecture method in teaching?
3. Is there a significant difference in the learners' problem-solving skills before and after their exposure to the heuristic method approach teaching and the lecture method in teaching?
4. Is there a significant difference in the mathematics performance of the learners before and after their exposure to the heuristic method of teaching and lecture method of teaching?

### **IV. THEORETICAL AND CONCEPTUAL FRAMEWORK**

This study was anchored on the theory of George Pólya's heuristic. He was a renowned Hungarian mathematician who contributed significantly to problem-solving with his heuristic approach. His work, particularly his book "How to Solve It," introduced a structured, four-step process that encourages critical thinking and fosters a deeper



understanding of the problem-solving process. George Pólya's heuristic problem-solving approach provides a robust framework for tackling challenges in various fields. By emphasising understanding, planning, execution, and reflection, this method encourages critical thinking, fosters a deeper understanding of the problem-solving process, and equips individuals with valuable skills applicable to various situations.

Herbert Simon, a Nobel laureate in economics, proposed the concept of bounded rationality, which directly relates to heuristics. According to Simon, heuristics are the mental shortcuts we use to navigate these limitations. They allow us to make "good enough" decisions, even when we do not have all the information or time for perfect analysis.

Pólya's problem-solving methodology consists of four steps: understanding the problem, devising a plan, executing the plan, and reflecting on the solution. Within this framework, he emphasises using various problem-solving strategies or heuristics. These strategies, such as drawing diagrams, making a list, working backwards, solving a more straightforward problem, or considering analogous problems, can be seen as heuristics because they offer practical guidelines or shortcuts for problem-solving.

However, it is essential to note that Pólya's work encompasses a broader problem-solving approach rather than a specific theory of heuristics. His methodology emphasises systematic thinking, logical reasoning, and metacognition alongside applying problem-solving strategies. Pólya aimed to provide a comprehensive framework for effective problem-solving rather than developing a specific theory dedicated to heuristics. In summary, while George Pólya's problem-solving strategies can be viewed as heuristics, his work does not present a dedicated theory of heuristics. Instead, his problem-solving methodologies offer practical guidelines and techniques within a broader problem-solving framework. George Pólya's work on problem-solving has made significant contributions to the field by providing practical strategies and a systematic framework for approaching and solving complex problems. While his work may not focus explicitly on the heuristics approach, his problem-solving strategies can be considered heuristics or mental shortcuts. Here are some key contributions of Pólya's work to problem-solving:

**1. Problem-solving methodology:** Pólya's framework provides a systematic problem-solving method. By emphasising steps such as understanding the problem, devising a plan, executing the plan, and reflecting on the solution, Pólya offers a structured approach that helps individuals navigate the problem-solving process effectively.

**2. Problem-solving strategies:** Pólya's strategies, such as drawing diagrams, making a list, working backwards, solving a more straightforward problem, and considering analogous problems, offer practical heuristics for problem-solving. These strategies serve as mental shortcuts or guidelines that

can be applied to various problem-solving contexts, enabling individuals to approach problems more efficiently and creatively.

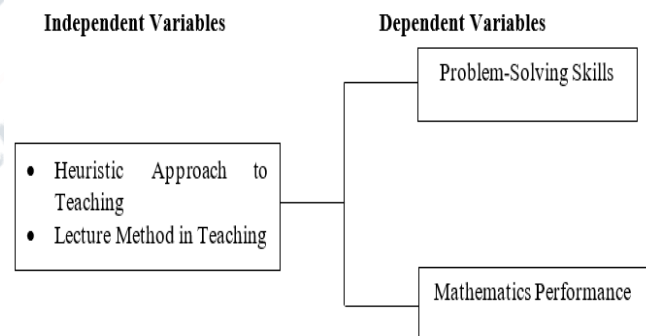
**3. Metacognition and reflection:** Pólya's emphasis on reflection and metacognition encourages individuals to think about their problem-solving processes. Individuals can gain insights into their thinking and problem-solving approaches by reflecting on their strategies, reasoning, and methods. This metacognitive aspect helps individuals refine their problem-solving skills and adapt them to different problem domains.

**4. Transferability of strategies:** Pólya's problem-solving strategies can be applied across various disciplines and problem domains. The author's heuristics are not limited to mathematics but can be utilised in engineering, medicine, business, and more.

**5. Educational impact:** Pólya's work has significantly influenced mathematics education. His problem-solving strategies and methodology have been integrated into curricula and instructional practices, helping students develop critical thinking skills and problem-solving abilities. Pólya's work has contributed to developing problem-solving-focused pedagogies and instructional approaches. George Pólya's work on problem-solving, though not solely focused on the heuristic approach, has profoundly impacted the field.

## V. CONCEPTUAL FRAMEWORK

The relationships among the variables used in this study are shown in Figure 3.



**Figure 1.** Paradigm showing the relationships among the variables in this study.

The study paradigm indicates how the heuristic approach affects the learners' problem-solving skills and mathematics performance. The independent variables constituted the strategy with and without a heuristic approach. In contrast, the dependent variables are the problem-solving skills and mathematics performance of the Grade 11 learners.

## VI. METHODOLOGY

### A. Research Design

The current quantitative studies use a quasi-experimental one-group pretest-posttest design to measure scores before and again following treatment and compare the difference between pretest and posttest scores (McCaleb, Anderson, and Hueston, 2008).

### B. Subjects of the Study

The study subjects are 80 Grade 11 learners from Pototan National Comprehensive High School, Pototan, Iloilo. The teacher-researcher would use the section where she teaches the subject to control the different stages of experimentation during the study's conduct.

### C. Research Instrument

To gather the data needed in this study, a researcher-made, validated, and pilot-tested "performance test" in mathematics was used. This research instrument consisted of a 10-item problem-solving test and a 50-item test on functions.

### D. Data Gathering Procedure

In the study, the researcher followed the matrix of activities with the following dates for one grading period and the topics to be included in the lessons for the six-week intervention: Furnish a copy of a letter approved by the principal; the lessons apply a heuristic approach and problem-solving method in teaching.

The experimental treatment would last 6 weeks and be divided into three stages: the pre-experimental stage, the experimental stage, and the post-experimental stage.

**Pre-experimental Stage.** The two strands from the Academic track were chosen to ensure they are almost identical in mathematics performance. The two classes are intact groups and would be randomly assigned to the heuristic approach for instruction and the non-heuristic approach for the instruction group by the flipped-coin method. This stage would be done before the start of the experiment. Both groups remained intact throughout the experimental period. Preparation of a heuristic approach to teaching problem-solving would be prepared for the experimental group. A pretest for three instruments would be administered to the two groups to determine the learners' performance, critical thinking and problem-solving skills in mathematics. The learners are given a 10-item problem-solving test with the corresponding scores of 50 points to determine their problem-solving skills and critical thinking using the heuristic approach. To check the work of learners, the researcher would ask for the help of co-teachers teaching mathematics in the same field, and they serve as inter-raters for the variations of scores following the given rubric. The data were gathered, computer-processed, tabulated, analysed, and interpreted using appropriate Statistics.

The researcher prepared lesson plans on every topic during

the experiment and a table of specifications (TOS) for the performance test. The initial draft of the Performance Test was submitted for face and content validation to a jury of three experts in mathematics and test construction. Upon the experts' approval, it will be pilot-tested with Grade 11 learners in other high schools.

**Experimental Stage.** The schedule below is followed throughout the experimental period:

Second to seventh week: Heuristic Approach for Instruction Group Academic Strand (HUMSS-Aguinaldo and Traditional Approach for Instruction Group Academic Strand (HUMSS-Mabini)

During the heuristic approach to instruction, the two groups were exposed to the same lessons and references; they differed only in the teaching approach to which they were subjected. The students in the experimental group are exposed to a heuristic approach to instruction. Since the intervention occurred only during the mathematics class, the heuristic approach for the instruction group will be utilised. The actual teaching is done over 6 weeks; in the first week, the researcher used a more straightforward case, the second week was computing and simplifying, the third week was guessing and checking, the fourth week was making a model or diagram, the fifth week was making a table, and the sixth week was looking for patterns. The researcher evaluated them afterwards to assess their learning from the said intervention. The learners are scored individually.

In the heuristic approach for instruction group, the teacher taught the whole class by using the approach to solve problems, worksheets, and sample mathematics problems.

Learners from this group also took the entire test, like those in the heuristic approach instruction group, and their responses were scored individually to test their critical and problem-solving skills. To control the teacher factor, the researcher handled and taught both groups. The teaching sessions were scheduled so that both groups were exposed to the same class period.

**Post-experimental stage.** At the end of the sixth-week period, the learners from both groups are given a posttest to determine their performance in the subject. The data gathered are computer-processed, tabulated, analysed, and interpreted using appropriate Statistics. For analysis, the researcher used the following scale of means and corresponding interpretations to determine the learners' performance in mathematics, both in the pretest and posttest.

### E. Data Analysis

After the experiment, the data gathered for this study were subjected to appropriate computer-processed statistics employing the Statistics Package for Social Sciences (SPSS) software. The level of significance was 0.05.

Frequency count, means, and standard deviations were used for descriptive statistics, and the t-test for dependent samples was used for inferential statistics.

## VII. RESULTS AND DISCUSSION

This section presents the findings of the study.

**Table I:** Pretest Problem-solving Skills of the Learners with and Without Heuristic Approach in Teaching

Category	n	Mean	Description	SD
With Heuristic Approach	40	10.80	Developing	2.88
Without Heuristic Approach	40	10.55	Developing	3.37

Note: Advanced (40.01-50.00), Proficient (30.01-40.00), Approaching Proficiency (20.01-30.00), Developing (10.00-20.00), Beginning (less than 10)

Table 1 presents the pretest problem-solving skills of the learners with and without a heuristic approach to teaching.

The results below showed that the pretest problem-solving skills of learners with and without a heuristic approach in teaching were “developing,” with a mean of 10.80 (SD = 2.88) and 10.55 (SD = 3.37). This means that both groups have the same level of problem-solving skills. This outcome is because the topics have not yet been explained to them. Simon (2021) has emphasised that heuristic methods can enhance cognitive engagement by requiring students to analyse, reason, and reflect upon various problem-solving strategies.

**Table II:** Posttest Problem-solving Skills of the Learners with and Without Heuristic Approach in Teaching

Category	n	Mean	Description	SD
With Heuristic Approach	40	27.75	Approaching proficiency	2.07
Without Heuristic Approach	40	26.85	Approaching proficiency	1.66

Note: Advanced (40.01-50.00), Proficient (30.01-40.00), Approaching Proficiency (20.01-30.00), Developing (10.00-20.00), Beginning (less than 10)

Table 2 presents the posttest problem-solving skills of the learners with and without a heuristic approach to teaching.

The data in Table 2 revealed that the posttest problem-solving skills of learners with and without the heuristic approach were “approaching proficiency”. with a mean of 27.75 (SD = 2.07) and 26.85 (SD = 1.66). The result shows an increase in the level of problem-solving skills in mathematics. This result means that teaching the learners with and without a heuristic approach improved their problem-solving skills. Kapur and Bielaczyc (2021) have emphasised that the heuristic approach can enhance cognitive engagement by requiring students to analyse, reason, and reflect upon various problem-solving strategies.

**Table III:** Pretest Mathematics Performance of the Learners with and Without Heuristic Approach in Teaching

Category	n	Mean	Description	SD
Mathematics Performance				
With Heuristic Approach	40	29.50	Approaching proficiency	4.08

Without Heuristic Approach	40	29.43	Approaching proficiency	3.37
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Note: Advance (40.01-50.00), Proficient (30.01-40.00), Approaching Proficiency 20.01-30.00), Developing (10.01-20.00), Beginning (less than 10)

Table 3 presents the pretest Mathematics Performance on General Mathematics of Grade 11 learners with and without Heuristic Approach.

The study revealed the pretest mathematics performance of learners with and without a heuristic approach in teaching, with results indicating "Approaching Proficiency" with a mean of 29.50 (SD = 4.08) and 29.43 (SD = 3.37), respectively. This result means that both groups have the same level of Mathematics Performance. The approaching proficiency obtained by the learners was reasonable enough because the topic had not yet been presented and discussed with them. The result of the study supported the study of Grootenboer (2020), which focused on General Mathematics concepts perceived as challenging to learn by Senior High School students—a study examining the influence of affective and background factors on mathematics performance.

**Table IV:** Posttest Mathematics Performance of the Learners with and Without Heuristic Approach in Teaching

Category	n	Mean	Description	SD
Mathematics Performance				
With Heuristic Approach	40	37.63	Proficient	5.08
Without Heuristic Approach	40	30.03	proficient	5.57

Note: Advance (40.01-50.00), Proficient (30.01-40.00), Approaching Proficiency 20.01-30.00), Developing (10.01-20.00), Beginning (less than 10)

Table 4 presents the posttest Mathematics Performance of learners with and without Heuristic Approach in teaching.

The study revealed the posttest mathematics performance of learners with and without the heuristic approach, with results indicating “Proficient” with a mean of 37.63(SD = 5.08); this improvement emphasises that heuristic teaching methods, which encourage exploration and problem-solving can enhance mathematical performance by fostering analytical skills and independent thinking. The group taught through lecture-based methods with a mean of 30.03 (SD = 5.57), achieving a "Proficient" status. However, this increase was less pronounced than in the heuristic group, supporting studies suggesting that while lectures can aid foundational understanding, they may not stimulate the same level of engagement and skill development as heuristic methods.

The results parallel the studies of Sawyer and Wagner (2023) and Kapur and Bielaczyc (2021), which support the effectiveness of heuristic methods in mathematics education. They indicate that exposure to problem-solving strategies improves learners' performance. Additionally, Schoenfeld (2022) has shown that heuristic methods enhance



engagement and more profound understanding, which is essential for mathematical task proficiency.

**Table V:** t-test Result of the Difference in Pretest Problem Solving Skills of the Learners with and Without Heuristic Approach.

Category	n	df	(Sig2-tailed)	Interpretation
Pretest With Heuristic Approach	80	78	0.722	Not Significant
Without Heuristic Approach				

Table 5 presents the t-test result of the difference in the pretest Problem-solving skills of the learners with and without the Heuristic Approach.

This study examines differences in learners' problem-solving skills before and after exposure to heuristic and lecture-based teaching methods, analysing the results for statistical significance. Results reveal no statistically significant difference between groups before and after intervention with the heuristic approach. The pretest scores for the group exposed to the heuristic and the lecture methods showed a significance value of  $p=0.722$  and ( $df = 78$ ), indicating a "Not Significant" difference in initial problem-solving skills.

**Table VI:** t-test Result of the Difference in Posttest Problem-solving Skills of the Learners with and Without Heuristic Approach.

Category	n	df	(Sig2-tailed)	Interpretation
Posttest With Heuristic Approach	80	78	0.415	Not Significant
Without Heuristic Approach				

Table 6 presents the t-test Result of the Difference in Posttest problem-solving skills of the learners with and without a Heuristic Approach.

Following the intervention, the posttest scores between these groups also yielded a non-significant result with  $p=0.415$  and ( $df = 78$ ), suggesting no substantial difference in problem-solving improvement between the heuristic and lecture methods.

The findings align with recent research, which suggests that while heuristic-based approaches are associated with improved problem-solving skills, their effects may not always yield statistically significant differences in skill acquisition compared to traditional methods when assessed over short periods (Sawyer & Wagner, 2023; Kapur & Bielaczyc, 2021).

**Table VII:** t-test Result of the Difference in Pretest Mathematics Performance of the Learners with and Without Heuristic Approach.

Category	n	df	(Sig2-tailed)	Interpretation
Pretest With Heuristic Approach	80	78	0.929	Not Significant
Without Heuristic Approach				

Table 7 presents the t-test result for the difference in the pretest general Mathematics performance of Grade 11 learners with and without a Heuristic Approach.

This study evaluates differences in the mathematics performance of learners before and after exposure to heuristic and lecture-based teaching methods, revealing a statistically significant improvement in the post-intervention scores for the heuristic method. Initially, mathematics performance between the groups showed no significant difference, with a pretest significance level of  $p=0.929$  and ( $df = 78$ ), indicating no statistically significant variation in initial math abilities.

**Table VIII:** t-test Result of the Difference in Posttest Mathematics Performance of the Learners with and Without Heuristic Approach.

Category	n	df	(Sig2-tailed)	Interpretation
Posttest With Heuristic Approach	80	78	0.000	Significant
Without Heuristic Approach				

Table 8 presents the t-test result of the difference in post-test mathematics performance between learners with and without the heuristic approach.

The data revealed that the post-intervention results showed a highly significant difference, with  $p=0.000$  and ( $df = 78$ ), demonstrating that the heuristic approach substantially improved mathematics performance compared to the lecture method.

These findings are consistent with recent research demonstrating the efficacy of heuristic teaching strategies in enhancing mathematical understanding and performance. Studies suggest that heuristic methods, which emphasise discovery and problem-solving, support deeper comprehension and retention of mathematical concepts, fostering more effective learning than traditional lecture methods (Kapur & Bielaczyc, 2021; Rittle-Johnson & Schneider, 2022).

## VIII. CONCLUSIONS AND RECOMMENDATIONS

Based on the findings and observations, the following conclusions were drawn:

The findings suggest that while heuristic and lecture-based

methods improve learners' problem-solving and mathematics performance, the heuristic approach significantly impacts problem-solving skills, particularly in supporting analytical thinking. Although initial skill levels were similar between groups, post-intervention results indicate that the heuristic method significantly boosts mathematics proficiency in math performance attributed to problem-solving skills. This result suggests that heuristic teaching reinforces mathematical abilities and fosters a strong link between problem-solving and critical thinking, supporting recent effectiveness research.

The findings indicate that heuristic and lecture methods improved learners' problem-solving and mathematics performance, moving from "Developing" to "Approaching Proficiency" in problem-solving skills. Mathematics performance improved across both methods, with the heuristic group reaching "Proficient" levels.

Based on the study's findings, several recommendations can be made to enhance the mathematics performance of Grade 11 learners and address the competencies of mathematics teachers. Curriculum planners may incorporate heuristic methods into mathematics curricula to develop problem-solving and critical-thinking skills effectively. Structured problem-solving activities may be embedded across topics to enhance the integration of these skills and their application in real-world scenarios. Mathematics teachers may adopt and practice heuristic teaching strategies to boost problem-solving and critical-thinking abilities among students. Regularly incorporate open-ended problems that challenge students' reasoning and analytical thinking. Evaluate the effectiveness of these strategies and adjust approaches based on learner feedback and performance. Future researchers may investigate the long-term effects of heuristic teaching on academic performance and cognitive skill development. Future studies could explore variations across subjects and age groups to understand the broader impact of heuristic methods.

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